

**Exercise 6**

Evaluate the line integral, where  $C$  is the given curve.

$$\int_C e^x dx, \quad C \text{ is the arc of the curve } x = y^3 \text{ from } (-1, -1) \text{ to } (1, 1)$$

**Solution**

Parameterize the curve by setting  $y = t$ , which then means  $x = t^3$ , and having  $-1 \leq t \leq 1$ . With this parameterization in  $t$ , the line integral becomes

$$\begin{aligned} \int_C e^x dx &= \int_{-1}^1 e^{x(t)} \frac{dx}{dt} dt \\ &= \int_{-1}^1 e^{t^3} (3t^2) dt. \end{aligned}$$

Make the following substitution.

$$\begin{aligned} u &= t^3 \\ du &= 3t^2 dt \end{aligned}$$

Therefore,

$$\begin{aligned} \int_C e^x dx &= \int_{(-1)^3}^{(1)^3} e^u du \\ &= \int_{-1}^1 e^u du \\ &= e^u \Big|_{-1}^1 \\ &= e - e^{-1}. \end{aligned}$$